Reasoning about Conditional Probability and Counterfactuals

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Abstract

The analysis of static games involves descriptions of beliefs about beliefs. When beliefs are probabilistic, this is modeled by Harsanyi type spaces, and more generally belief spaces, where beliefs vary with states. Belief spaces were characterized axiomatically using operators of the form "John's probability of x is at least p". The analysis of dynamic games requires conditional belief systems, and in particular conditional beliefs about conditional beliefs. In this paper we consider spaces where conditional belief systems vary with states, and we axiomatize such spaces using conditional belief operators of the form "John's probability of x given y is at least p". An informal assumption of probability theory is that the agent is being informed of the conditioning event. In our model this can be made formal, as being informed, or being certain of an event is itself an event in the model. Using the axiom of *Echo*, which appears in many guises in the theory of belief spaces, we relate conditional and unconditional probabilities: at each state, John's conditional probability of x given that he is certain of y is an average of the unconditional beliefs he may have when he is certain of y. Our operators naturally define a kind of counterfactual implication that satisfies the usual axioms behind the standard models of counterfactuals due to e.g. Lewis and Stalnaker.